# ANTI–SAG BARS INFLUENCE ON THE LATERAL BUCKLING RESISTANCE OF PURLINS RESTRAINED BY SHEETING

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## ABSTRACT

*Cold-formed purlins may increase considerably their stiffness and resistance if they are effectively restrained by profiled steel sheeting that provides full lateral restraint to the top flange of the purlin through the troughs of the sheet.*

*Despite the improvement of both the purlin stability and the partial twisting restrain given by steel sheeting, where the free flange is in compression due to in-plane bending the purlin resistance can fall due to the lateral buckling of the unrestrained flange.*

*The stability of the free flange within zones in compression – at internal supports under gravity loading and in the spans for uplift loading – can be maintained through the use of one or plus anti-sag bars per span.*

*Regardless of the computing methods available, the difficulties arisen within the buckling lengths calculation for the compressed free flange and the verification of its stability between two anti-sag bars, have often driven to develop analytical methods, such as that included in Eurocode 3-1.3, that do not solve the full casuistic resulting from the combination of multiple anti-sag bars with multiple spans cases under several types of loading.*

*The state-of-the-art on this point is hereby analysed in order to detect non-solved cases and study alternative design methods.*

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## 1.– INTRODUCTION

Within the design process of cold-formed purlins, a significant improvement of their behaviour can be taken into account if trapezoidal steel sheeting is continuously connected to the top flange of the purlin through the troughs of the sheets (see figures 1 and 2).


Fig. 1 – Cold-formed purlins restrained by sheeting

The effective connection of the top flange of the purlin to the steel sheeting increases considerably its stiffness and resistance since full continuous lateral restraint is given. Despite the improvement of both the purlin stability and the partial twisting restraint given by steel sheeting, where the free flange is in compression due to in-plane bending the purlin resistance can fall due to the lateral buckling of the unrestrained flange (see figure 3).

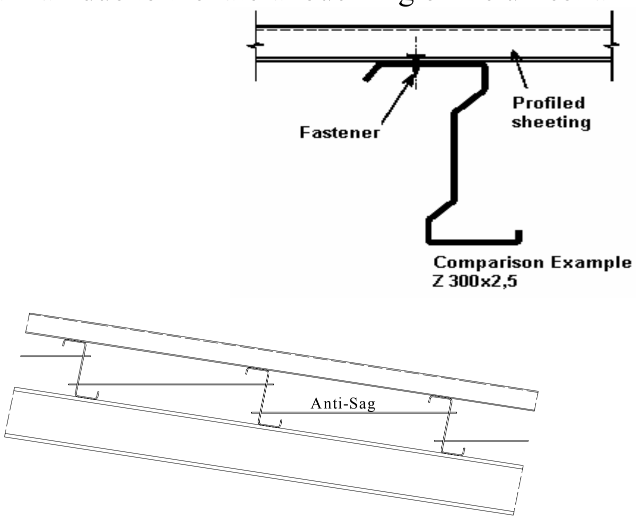



Fig. 2 – Zeta Purlin restrained by profiled (trapezoidal) sheeting and Anti-Sag bars

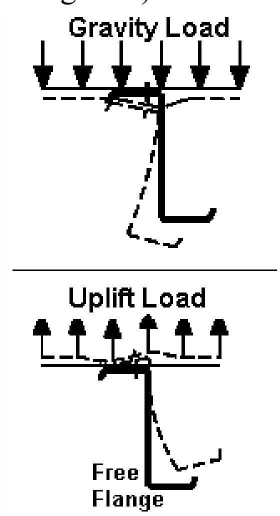



Fig. 3 – Lateral buckling of unrestrained flange

The stability of the free flange within zones in compression – at internal supports under gravity loading and in the spans for uplift loading – can be maintained through the use of one or plus anti-sag bars per span.

Within the design through calculation of a purlin restrained by sheeting, unless a second order analysis is carried out designer must use a calculation method that takes into account the above mentioned tendency of the free flange to move laterally – see again figure 3 – (and the additional stresses induced by this movement). In that sense, **Eurocode 3-1.3** ([1]) proportionates the following expressions to determine the buckling resistance of the free flange at zones in compression:

$$\frac{1}{\chi}\cdot\left[\frac{M_{y.Sd}}{W_{eff.y}}+\frac{N_{Sd}}{A_{eff}}\right]+\frac{M_{fz.Sd}}{W_{fz}}\leq f_{yb}/\gamma_{M1} \qquad (1)$$

in which χ is the reduction factor for flexural buckling of the free flange, obtained using the buckling curve *a* for the relative slenderness $\overline{\lambda_{fz}}$ resulting from:

$$\chi=\frac{1}{\Phi+\sqrt{\Phi^2-\overline{\lambda}^2}}\ ,\quad but\ \chi\leq 1.0 \qquad (2)$$

$$\Phi=0.5\cdot\left[1+0.21\cdot(\overline{\lambda}_{fz}-0.2)+\overline{\lambda}_{fz}^2\right],\ and \qquad (3)$$

$$\overline{\lambda_{fz}}=\frac{l_{fz}/i_{fz}}{\lambda_1}\ ,\quad with \qquad (4)$$

$$\lambda_1=\pi\cdot\left[E/f_{yb}\right]^{0.5} \qquad (5)$$

where $l_{fz}$ is the buckling length for the free flange obtained from:

$$l_{fz}=\eta_1\cdot L_a\cdot(1+\eta_2\cdot R^{\eta_3})^{\eta_4} \qquad (6)$$

where, $L_a$ is the distance between anti-sag bars, or if none are present, the span L of the purlin

$$and\quad R=\frac{K\cdot L_a^4}{\pi^4\cdot E\cdot I_{fz}} \qquad (7)$$

where $I_{fz}$ is the second moment of area of the gross cross-section of the free flange plus 1/6 of the web height, for bending about the z-z axis,

and K is the lateral spring stiffness per unit length

$M_{fz.Sd}$ is the lateral bending moment that should be determined from:

$$M_{fz.Sd}=\beta_R\cdot k_h\cdot\delta\cdot q_{Fd}\cdot L_a^2\ ,\ where\ \beta_R\ is\ a\ correction\ factor\ for\ the\ effective\ spring\ support,\ given\ within\ EC3-1.3\ tables \qquad (8)$$

## 2.– STATE-OF-THE-ART

On next paragraphs a wide scope analysis of the state-of-the-art on the buckling lengths calculation for the compressed free flange and on the verification of its stability between two anti-sag bars is developed in order to detect non-solved cases and study alternative design methods. Many bibliographic references are included so readers will be able to increase their knowledge on any of the subjects treated herein.

## 2.1.– Eurocode 3-1.3: 1996 ([1])

The coefficients $\eta_1$ to $\eta_4$ needed for the calculation of the buckling length are obtained, using the method given by **Eurocode 3-1.3**, from the following table:

Table 1 – Eurocode values for the coefficients needed for the Free Flange Buckling Length calculation

| Number of anti-sag bars per span | $\eta_1$ | $\eta_2$ | $\eta_3$ | $\eta_4$ |
|---|---|---|---|---|
| 0 | 0.526 | 22.8 | 2.12 | -0.108 |
| 1 | 0.622 | 66.7 | 2.68 | -0.084 |
| 2 or 3 | 0.713 | 62.7 | 2.75 | -0.084 |
| More than 3 | 1.000 | 30.4 | 2.28 | -0.108 |

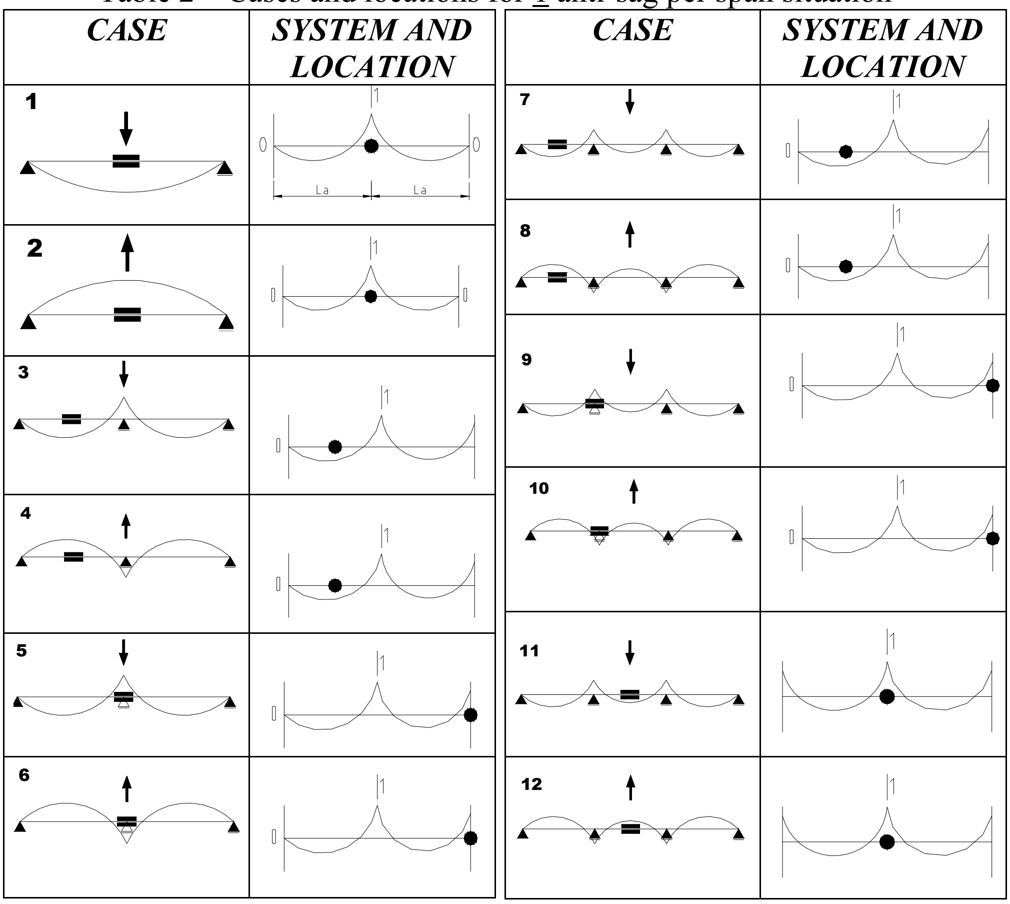
For gravity loading, if there are more than three equally spaced anti-sag bars, the buckling length need not be taken as greater than the value for two anti-sag bars, with $L_a$=L/3.

For uplift loading, provided that 0 <= $R_0$ <= 200, the buckling length of free flange for variations of the compressive stress over the length $L_0$ may be obtained from:

$$l_{fz} = 0.7 \cdot L_0 \cdot (1+13.1 \cdot R_0^{1.6})^{-0.125} \quad (9)$$

The whole set of checking cases that may have to be done within a purlin design process is sumed up on the next table, where each of the twelve cases is associated at each number of anti-sags per span to a certain set of values of the $\eta_i$ coefficients mentioned above.

Table 2 – Cases and locations for 1 anti-sag per span situation



Within the design method exposed above, included in **Eurocode 3-1.3**, there are cases which are still non solved, such as that corresponding to a purlin under uplift load and with one anti-sag per span. These cases may be solved through extrapolation of equations given for similar situations but this conservative calculation results in non economic solutions.

In order to show the set of results that are obtained from the appliance of Eurocode design method to the example purlin shown on figure 2, the following figures are shown:

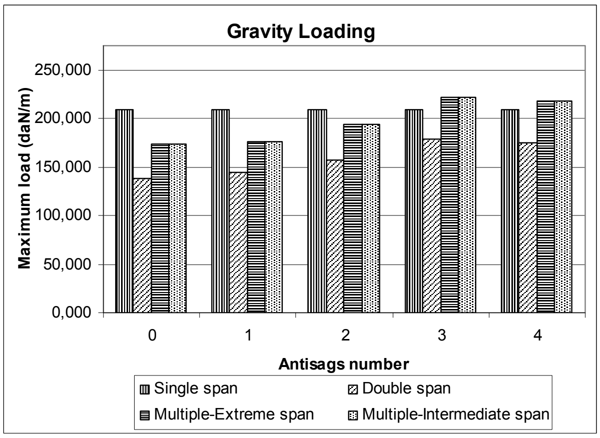



Fig. 4 – EC3-1.3 Maximum load for gravity loading

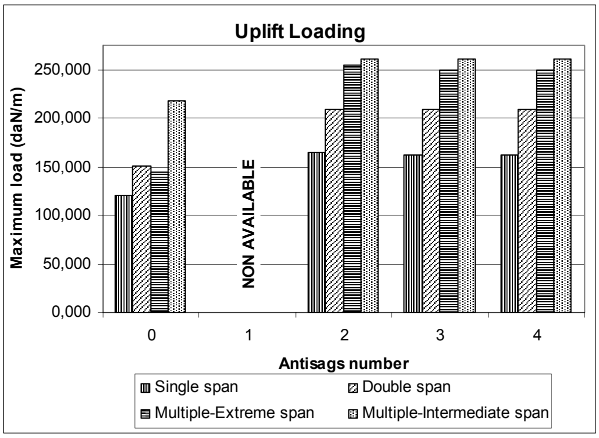



Fig. 5 – EC3-1.3 Maximum load for uplift loading

Figures 4 and 5 show the maximum load obtained from equation (1) applying the set of $\eta_i$ coefficients given by Eurocode and shown in Table 1. As it can be clearly observed, there are certain cases still non solved within the method given as it has already been mentioned. There are also some incoherencies since, for instance, in case under gravity loading the maximum load corresponding to a double or a multiple span purlin is smaller than that corresponding to a single span one (this paradoxal result would be avoided by means of a global plastic analysis).

Moreover, under gravity loading when the coefficient R – see equation (7) – approaches to zero conservatism in Eurocode 3 design method arises for multiple span cases while uncorservatism arises when R equals 20 to 25. In the meanwhile, under uplift loading the critical length equation given in **Eurocode 3-1.3** is suitable for simply supported single span purlins but gives conservative approximations for two or more span purlins.

## 2.2.– Leopold Sokol ([2])

New approaches have been later developed in order to proportionate an universal method useful for all of the cases while improving the behavioural model. In that sense, **Leopold Sokol** ([2]) developed a general approach to determine the coefficients $\eta_i$ that appear in the empiric stability equation (6). This proposal, developed using the Galerkin method, is shown on tables 3 and 4:

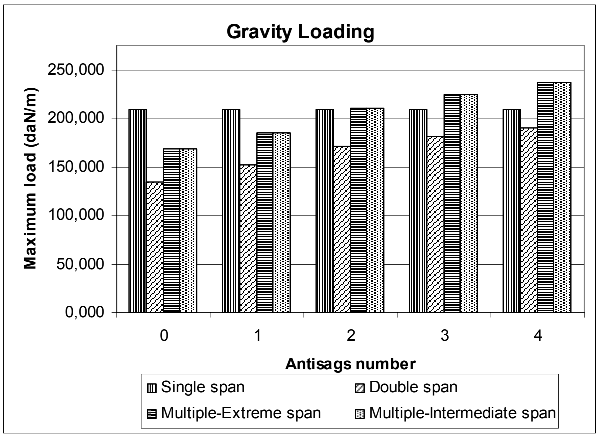
Table 3 - **Sokol** values for the coefficients needed for the Free Flange Buckling Length calculation (Table I of II)

| | **Single span** | | | | | **Multiple span** | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | **End span** | | | | | | | | | | **Intermediate span** | | | | | | | | | |
| **Load** | **Uplift** | | | | | **Uplift** | | | | | **Gravity** | | | | | **Uplift** | | | | | **Gravity** | | | | |
| **Antisag** | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 4 |
| **Curve** | 1 | 2 | 5 | 5 | 5 | 7 | 2 | 2 | 5 | 5 | 4 | 7 | 6 | 1 | 1 | 3 | 7 | 2 | 2 | 2 | 8 | 6 | 6 | 1 | 1 |

Table 4 - **Sokol** values for the coefficients needed for the Free Flange Buckling Length calculation (Table II of II)

| **Curve** | $\eta_1$ | $\eta_2$ | $\eta_3$ | $\eta_4$ |
|---|---|---|---|---|
| **1** | 0,6938 | 5,4463 | 1,2733 | -0,1680 |
| **2** | 0,8000 | 6,7481 | 1,4843 | -0,1545 |
| **3** | 0,3059 | 0,2323 | 0,7424 | -0,2790 |
| **4** | 0,4139 | 1,7218 | 1,1073 | -0,1782 |
| **5** | 0,9021 | 8,5456 | 2,1783 | -0,1108 |
| **6** | 0,5956 | 2,3250 | 1,1484 | -0,1920 |
| **7** | 0,5144 | 1,2545 | 0,8679 | -0,2415 |
| **8** | 0,6574 | 8,1695 | 2,2210 | -0,1071 |

In order to show the set of results that are obtained from the appliance of **Sokol** coefficients to the **Eurocode 3-1.3** design method in case of a certain purlin, the following figures are shown:



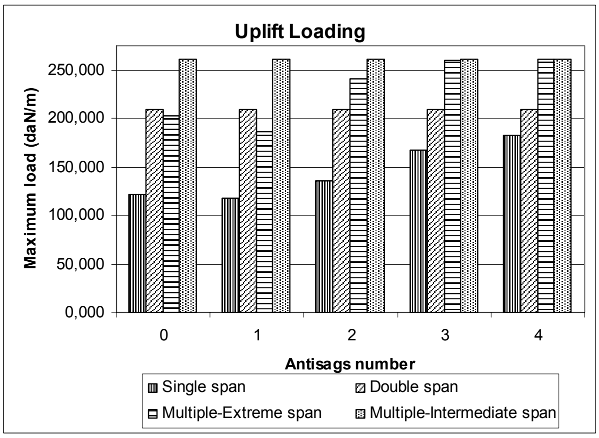




Fig.6 – EC3-1.3 Maximum load for gravity loading with Sokol coefficients

Fig.7 – EC3-1.3 Maximum load for uplift loading with Sokol coefficients

As it can be observed in figures 6 and 7, Sokol coefficients $\eta_i$ provide a full solution that covers all cases non available using the **Eurocode 3-1.3** values exposed on Table 1. However, there are still some incoherencies resulting from the use of Eurocode 3 design method applying these coefficients, such as the decrease of the maximum load that is obtained under uplift loading considering one anti-sag per span with respect to that without anti-sags.

**2.3.– S. Baraka** ([3])

**S. Baraka** published a new approach based on numerical techniques in combination with the finite elements method. It consists on a finite elements program that provides the buckling length values for cases non included within earlier methods, such as that under a compressive forces distribution, cases without lateral supports at points vertically supported or with non equidistant anti-sags.

On next figures are shown the results of applying this last method to calculate the free flange buckling length needed to calculate its buckling resistance by means of equation (1) – note that $L_a$ is the distance between anti-sag bars, or if none are present, the span L of the purlin and R is the coefficient of the spring support given by equation (7) –:

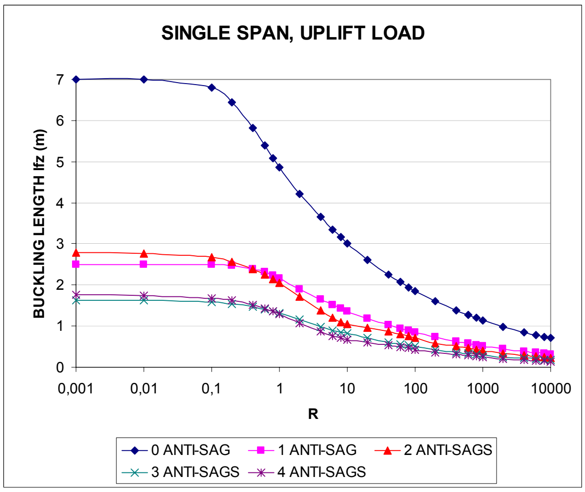


Fig. 8 – Single span, uplift load, Buckling length

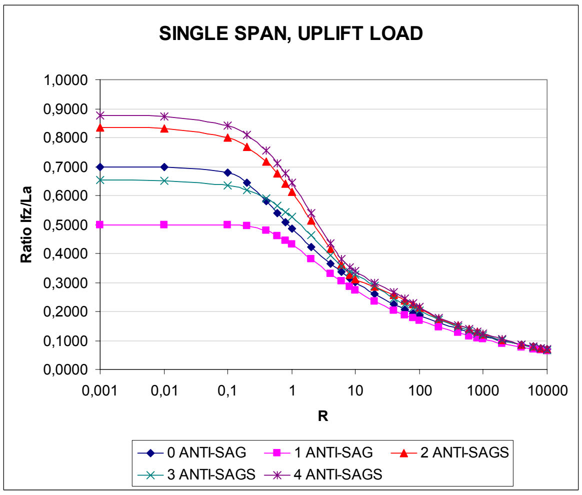


Fig. 9 – Single span, uplift load, Ratio lfz/La

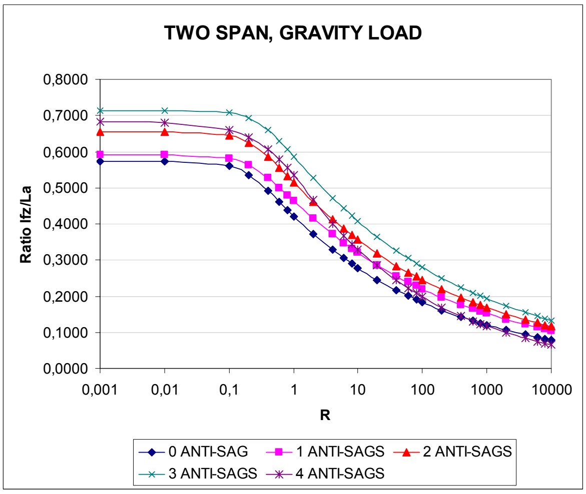


Fig. 10 – Two span, gravity load, Ratio lfz/La

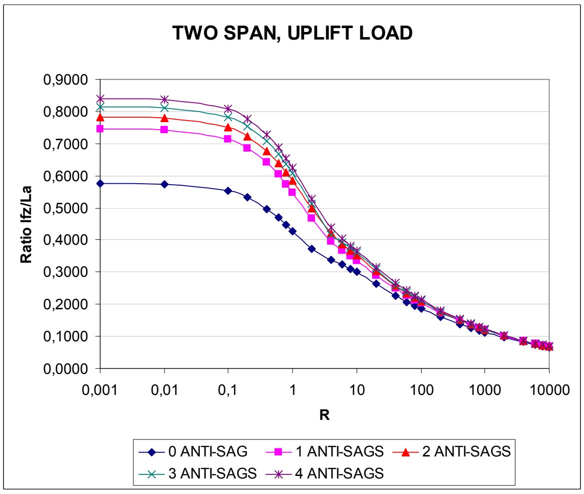


Fig. 11 – Two span, uplift load, Ratio lfz/La

The curves shown in figures 9, 10 and 11 can be expressed through regression technique in equations analogue to equation (6). For *instance*, in case of a two span purlin under gravity loading (figure 10), the $\eta_i$ coefficients that would represent the curves shown are the following:

Table 5 - **Baraka** values for the coefficients needed for the Free Flange Buckling Length calculation in two span case for gravity loading

| **Anti-Sags per span** | $\eta_1$ | $\eta_2$ | $\eta_3$ | $\eta_4$ |
|---|---|---|---|---|
| **0** | 0.574 | 28.008 | 2.000 | -0.091 |
| **1** | 0.590 | 19.130 | 2.000 | -0.081 |
| **2** | 0.654 | 18525 | 2.000 | -0.081 |
| **3** | 0.713 | 10.047 | 2.000 | -0.081 |

However, from the analysis of figure 8 the following observation is derived: For a single span case under uplift loading, at the low range of values of the coefficient of the spring support R (this is from zero to one) the buckling length corresponding to the 1 anti-sag per span situation is smaller than that corresponding to the 2 anti-sag per span situation, and similarly occurs in cases of three and four ant-sags per span. Analogue considerations are derived from the three span cases.

Herein, it must be noted that the range of values of R corresponding to most of the commercial types of purlins available and in real design conditions, vary between 0 and 25, decreasing fast as the number of anti-sags per span increases.

### 2.4.– Eurocode 3-1.3:20xx 30 April 2002 ([4])

As this paper was being written, a new review of the Eurocode 3-1.3 appeared as a 2nd draft with the denomination *Eurocode 3-1.3:20xx 30 April 2002*. The improvements included within the new text were significant enough to be taken into account in the analysis developed herein.

The main feature of the improvements introduced by the Eurocode 3-1.3 writing comission was the adoption of the sets of coefficients provided some years before by **Leopold Sokol** ([2]) – shown on paragraph 2.2 of this paper –, coefficients which this second draft present grouped in the following tables 6 and 7 for gravity and uplift load respectively.

Table 6 – **Eurocode 3-1.3:20xx** values for the coefficients needed for the Free Flange Buckling Length calculation under gravity load (Table I of II)

| Situation | Anti sag-bar Number | $\eta_1$ | $\eta_2$ | $\eta_3$ | $\eta_4$ |
|---|---|---|---|---|---|
| End span | 0 | 0.414 | 1.72 | 1.11 | -0.178 |
| Intermediate span | | 0.657 | 8.17 | 2.22 | -0.107 |
| End span | 1 | 0.515 | 1.26 | 0.868 | -0.242 |
| Intermediate span | | 0.596 | 2.33 | 1.15 | -0.192 |
| End and intermediate span | 2 | 0.596 | 2.33 | 1.15 | -0.192 |
| End and intermediate span | 3 and 4 | 0.694 | 5.45 | 1.27 | -0.168 |

Table 7 – **Eurocode 3-1.3:20xx** values for the coefficients needed for the Free Flange Buckling Length calculation under uplift load (Table II of II)

| Situation | Anti sag-bar Number | $\eta_1$ | $\eta_2$ | $\eta_3$ | $\eta_4$ |
|---|---|---|---|---|---|
| Simple span | 0 | 0.694 | 5.45 | 1.27 | -0.168 |
| End span | | 0.515 | 1.26 | 0.868 | -0.242 |
| Intermediate span | | 0.306 | 0.232 | 0.742 | -0.279 |
| Simple and end spans | 1 | 0.800 | 6.75 | 1.49 | -0.155 |
| Intermediate span | | 0.515 | 1.26 | 0.868 | -0.242 |
| Simple span | 2 | 0.902 | 8.55 | 2.18 | -0.111 |
| End and intermediate spans | | 0.800 | 6.75 | 1.49 | -0.155 |
| Simple and end spans | 3 and 4 | 0.902 | 8.55 | 2.18 | -0.111 |
| Intermediate span | | 0.800 | 6.75 | 1.49 | -0.155 |

An additional modification consists on the establishment of a limit value at 40 for the R coefficient to accept the validity of the formulation given at Eurocode 3-1.3 for the correction factor $\beta_R$ that appears on equation (8).

Therefore, the same analysis previously done for **Leopold Sokol** ([2]) set of coefficients – paragraph 2.2 – can be developed for those included in the 2nd draft of Eurocode 3-1.3.

## 3.– PROPOSALS COMPARISON

Despite the difficulties found applying any of the sets of coefficients exposed above, including the approach developed on late sixties by **Timoshenko** ([5]) for the calculation of the buckling length, the comparison of the buckling length values obtained through all of these methods provides full information as it will be described on next figures and, in the meanwhile, makes easier the analysis of the state-of-the-art about the subject of this paper:

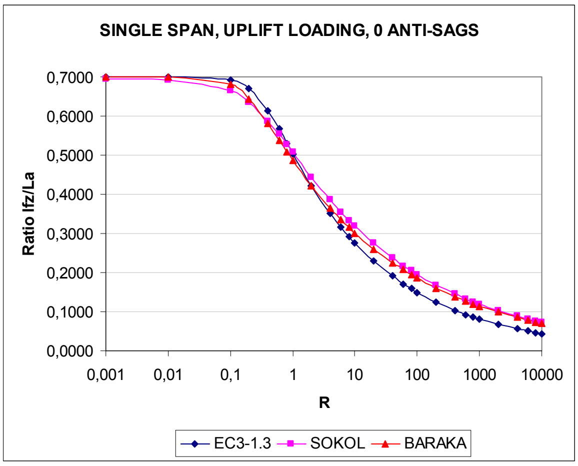



Fig.12 – Single span, uplift loading, 0 Anti-Sags comparison

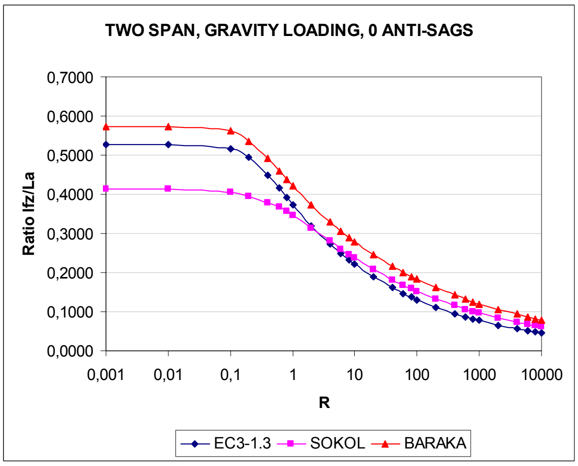



Fig.13 – Two span, gravity loading, 0 Anti-Sags comparison

Figure 13 shows that under gravity loading, when the coefficient R – see equation (7) – approaches to zero, conservatism in Eurocode 3-1.3 design method arises for multiple span cases while uncorservatism arises when R equals 20 to 25 (this conservatism must be understood both in absolute and in comparison terms). In the meanwhile, under uplift loading the critical length equation given in **Eurocode 3-1.3** is suitable for simply supported single span purlins but gives conservative approximations for two or more span purlins as it will be seen on figures 12 and 14 (see next page).

Beyond the higher or lower conservatism that the above analysed proposals introduce within the design of a cold-formed purlin restrained by sheeting, the difficulties found for their use are shown more clearly drawing the values of the reduction factor for flexural buckling of the free flange, named $\chi$ at equations (1) and (2), as a function of the parameter $R_L$ defined as the coefficient R – see equation (7) – multiplied by the term $(n+1)^4$, where n is the number of anti-sags per span. This drawing is shown on figure 15.

Where $R_L$ is defined in this way, a unique value of $R_L$ corresponds to a certain purlin (in this case the Zeta Purlin 300x2.5 mentioned on figure 2), on the contrary to what happened with the parameter R whose values depended on the number of anti-sags per span. This arrangement proportionates a simple graphic that shows inmediately the incoherency arising of the formulation previously exposed:

$$\chi_{0\ \text{Anti-Sags}} = 0.6563$$
$$\chi_{1\ \text{Anti-Sag}} = 0.5968$$
$$\chi_{2\ \text{Anti-Sags}} = 0.7517$$
$$\chi_{3\ \text{Anti-Sags}} = 0.8153$$
$$\chi_{4\ \text{Anti-Sags}} = 0.8854$$

As it can be seen, the reduction factor obtained for the case with 1 Anti-Sag per span is even lower than that obtained without Anti-Sags, result that is not reasonable.

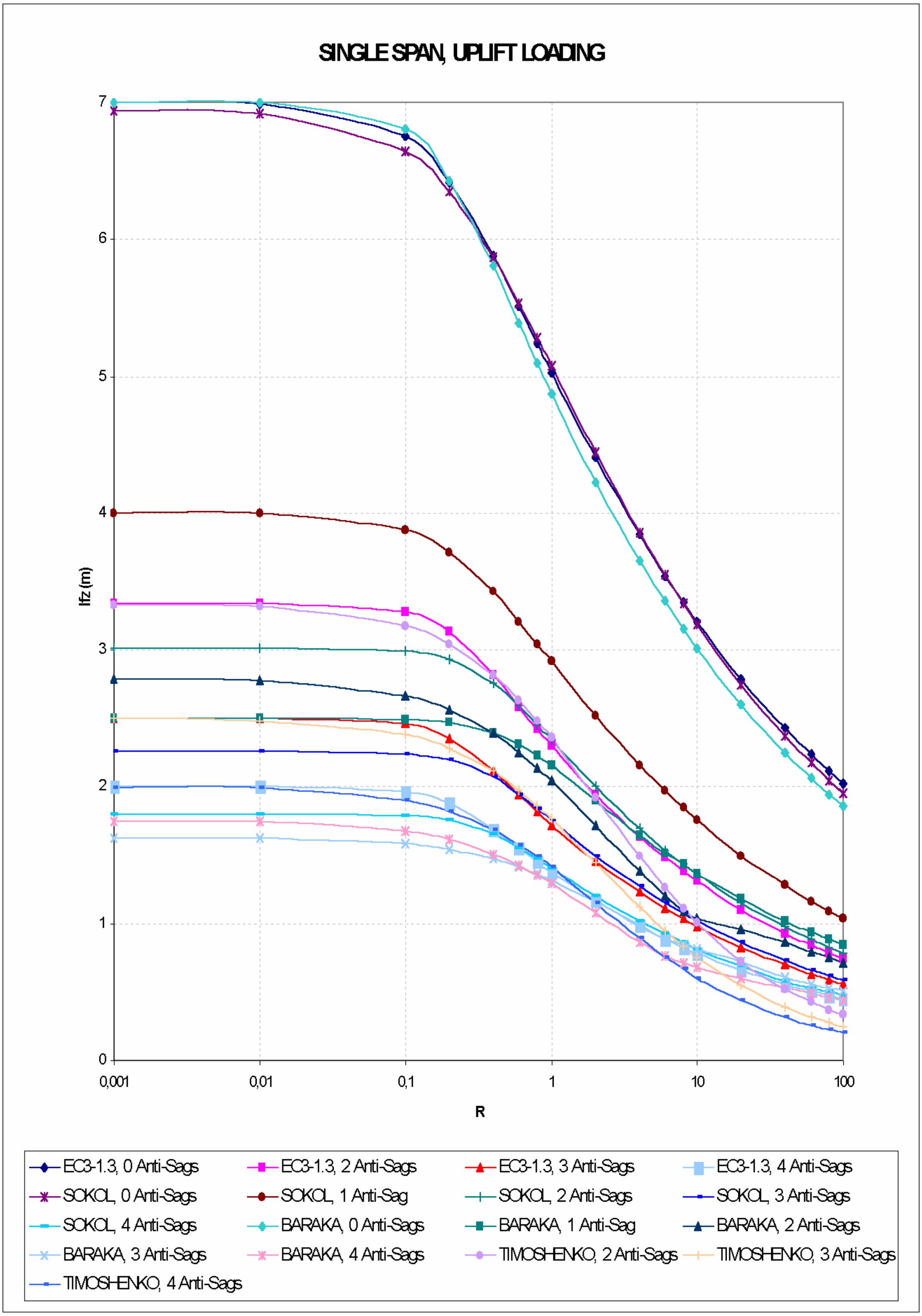



Fig.14 – Single span, uplift loading comparison

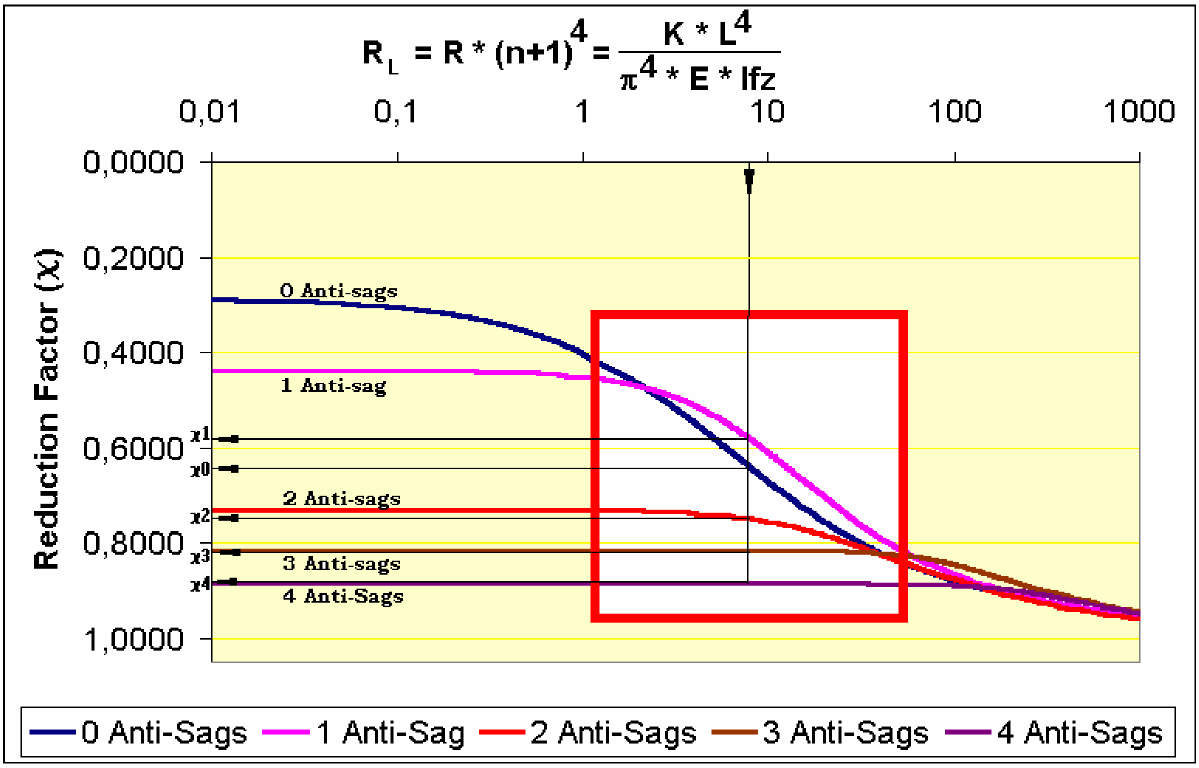



Fig. 15 – Reduction factor for flexural buckling of the free flange,χ, as a function of R

The paradoxal results shown on this figure, obtained for a 10 meters purling continuous over three spans under uplift load, also appear in many other cases.

## 4.– ABRIDGED CONCLUSSIONS

In this paper, the state-of-the-art on the buckling resistance calculation for the free flange of a cold-formed purlin restrained by sheeting has been analysed in order to detect non-solved cases and incoherencies arosen from the use of the latest regulating proposals on this subject.

The inducing cause of the incoherencies found may arise from the assumption done for the proposals development consisting on consider the ends of the span between two contiguous anti-sags as inflexion points of the free flange buckled shape.

## 5.– REFERENCES


[1] EUROCODE 3 *ENV 1993:1996 Design of steel structures – Part 1.3: General rules – Supplementary rules for cold formed thing gauge members and sheeting.*

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[3] Baraka S., *Flambement en milieu élastique d'une barre soumise à une distribution non uniforme d'efforts normaux*, Revue Construction Métallique, nº 3, 1996.

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